*TERMINAL REPORT*

# *Determination of the Error Cost Functional Used by Human Operators While Controlling Certain Compensatory Systems*

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# I. SUMMARY

The purpose of this research was to determine the error cost functional used by human operators while controlling low order compensatory control systems. The systems investigated were limited to those cases where the model of the forward loop of the control system -- including human operator and controlled element -- was satisfactorily represented by the McRuer crossover model. This forward loop model consists entirely of a gain K, a time-delay $\tau$, and a single integration. This simple model is applicable for a fairly wide range of low order controlled elements.

It was shown that when the crossover model is applicable, the two model parameters, K and $\tau$, are effectively adjusted to minimize the cost functional

$$J = \overline{e^2(t)} + \alpha \overline{c^2(t)}$$

with the constraint

$$\tau = \beta K, \quad 0.01 \leq \beta \leq 0.12 \text{ seconds}^2.$$

$\overline{e^2}$ is the mean squared control system error and $\overline{c^2}$ is the mean squared value of control effort. $\beta$ is inversely proportional to a subject's closed loop bandwidth, and thus varies between subjects, and even within the same subject at various stages of training. For a given control system, $\alpha$ is a constant that doesn't change either between subjects or

with subject practice. However, $\alpha$ does change with the type of system being controlled and becomes smaller as the control task becomes more difficult.

The conclusion is drawn that all subjects optimize the same cost functional when given the same control task. They perform differently, however, due to their varying abilities in processing high frequency signals. The subjects penalize control effort to a greater extent on easier tasks, than on harder tasks.

## II. FINAL REPORT

### A. Introduction

Since the start of World War II interest in manual control systems has been rapidly increasing [1]. A considerable amount of the research done in this area has been aimed at determining the characteristics of the human operator while he is functioning as the controller in a basic compensatory control task [2]. In this type of task the operator is placed into a control system described by the block diagram in Figure 1. The signal N(t) is Gaussian white noise, and the input filter is low pass with a cut-off frequency of 1 to 4 radians per second. The controlled element, $Y_c(s)$, is of low order, usually second or less.

Through the efforts of many different researchers, and by such diverse experimental techniques as random input describing functions [3], parameter tracking [4,5], and orthogonal filtering [6], several facts pertaining to the human operator have been verified. One of the more important of these findings, and one which is directly related to the research being reported, involves the human operator's describing function. For low order controlled elements (second or less), the human operator adjusts his mode of operation so that the entire forward loop of the compensatory system has the form [7]

$$\frac{\theta_0}{e}(s) = \frac{K\varepsilon^{-\tau s}}{s} .$$

It appears that whenever the system characteristics are changed, the human operator immediately changes his describing function so that the forward-loop describing function retains the form given above. The

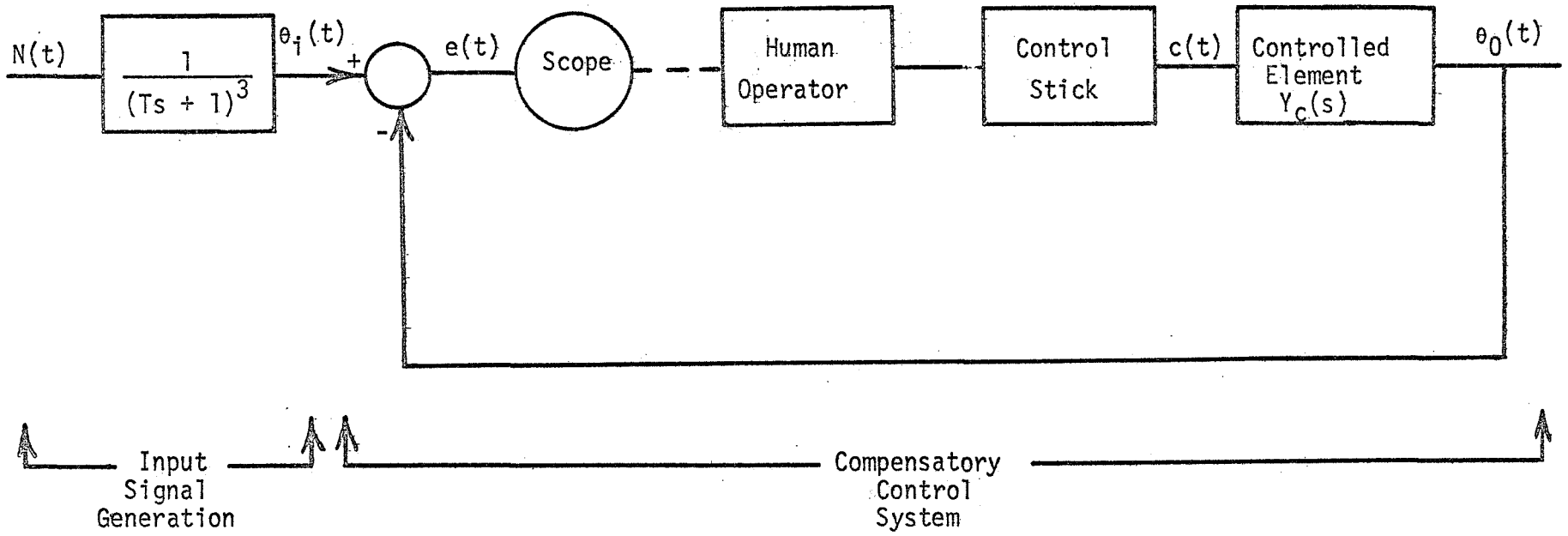



Figure 1 Typical Compensatory Tracking Task

magnitudes of K and $\tau$ depend upon $\theta_i(t)$, the controlled element, and the subject being tested.

This simple forward-loop model, called the "crossover model" by McRuer et. al. [7], fits the human operator describing function data extremely well over that portion of the frequency spectrum where input power is significant, and in the region where the absolute magnitude of the forward-loop describing function is unity. Since this latter region mainly determines closed-loop response, the closed-loop crossover model is a very realistic approximation to the compensatory system.

In 1967 Jackson [8] published the results of a parameter tracking study that indicated the compensatory control system of Figure 1 could be adequately modeled by the approximate crossover model of Figure 2. In this study the parameters K and $\tau$ were directly measured, and were found to change in definite patterns as a function of input badnwidth, subject practice, and order of the controlled element [9].

## B. Purpose of the Present Research

In pre-test orientation, the subjects to be tested in a compensatory control task are simply told to "keep the error e(t) as small as possible at all times." The question investigated in this research was: Exactly what cost functional is the human trying to minimize when he "keeps the error as small as possible?" It is natural to assume that he is optimum in some sense, or at least trying to be. The present research is intended to determine this optimizing functional, and to determine how this functional can be used in both the mathematical description of the human operator's control actions, and in manual control system design.

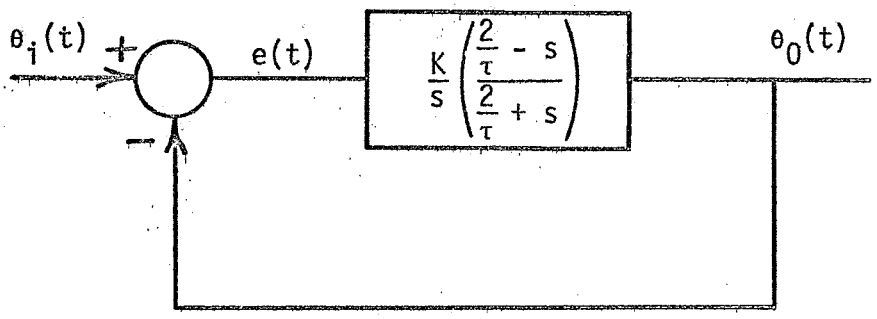



Figure 2 Approximate Crossover Model of The Compensatory System

There appear to be few results in the literature that are directly related to this problem.

(1) McRuer, et. al. [6,10] have postulated that the human minimizes the mean squared error subject to stability constraints on the compensatory control system. However, the constraints are given verbally in very general terms so that no mathematical form for the minimizing functional and associated constraints is given.

(2) It has been shown [11] that for Gaussian input systems when the mean squared error is minimized several other error scores are also minimized, indicating that the determination of a unique minimizing functional is probably not possible.

(3) Systems Technology, Inc. (STI) recently published a summary of new techniques which they hope to use in determining the functional which shows why the form of the crossover model is used by the operator [12].

The present research is different from the STI approach in that it is assumed from the start that the approximate crossover model is a good compensatory system model. (In fact, the only cases analyzed were those in which the approximate crossover model had proven to be exceptionally good.) The optimal policy sought was that policy which explained: why particular subjects adjust the K and $\tau$ parameters to the values they do; why K and $\tau$ change with input bandwidth; why K and $\tau$ differ between subjects; and why K and $\tau$ change with practice.

The data used in this analysis are from previous tests outlined in detail in reference [8].

C. A Unique Property of K and $\tau$.

In the parameter tracking tests reported in reference [8], the K and $\tau$ paramaters of the model of Figure 2 were directly measured. This was done for six subjects: three controlling a 5/s element and three controlling a $5/s^2$ element. In each case input filter time constants of 1, 1/2, and 1/4 seconds were all used, and K and $\tau$ values determined for each condition during each day of training.

One important property of these data, which was not discussed in the previous analysis, is that in the cases where the approximate crossover model was exceptionally good*, K and $\tau$ are highly correlated. This correlation exists at a given test condition, between subjects and even for the same subject on different days during the training period.

In Tables 1 and 2 in the Appendix the daily average values of K and $\tau$ are given for each subject at each of the test conditions where the approximate crossover model was exceptionally good. These test conditions are

(1) $Y_c(s) = 5/s$, T = 1 second,

(2) $Y_c(s) = 5/s$, T = 1/2 second,

(3) $Y_c(s) = 5/s^2$, T = 1 second.

In Figure 3 the data points for these conditions are plotted along with the linear regression line for each condition. The correlation coefficients are also given in the figure.

*"Exceptionally good" is taken to be those cases where the power in the error between model output and compensatory system output is less than 10% of the power out of the compensatory system. The Power Match [8] is thus > 90%. This indicates that the model is accurate and that the human operator is fairly linear.

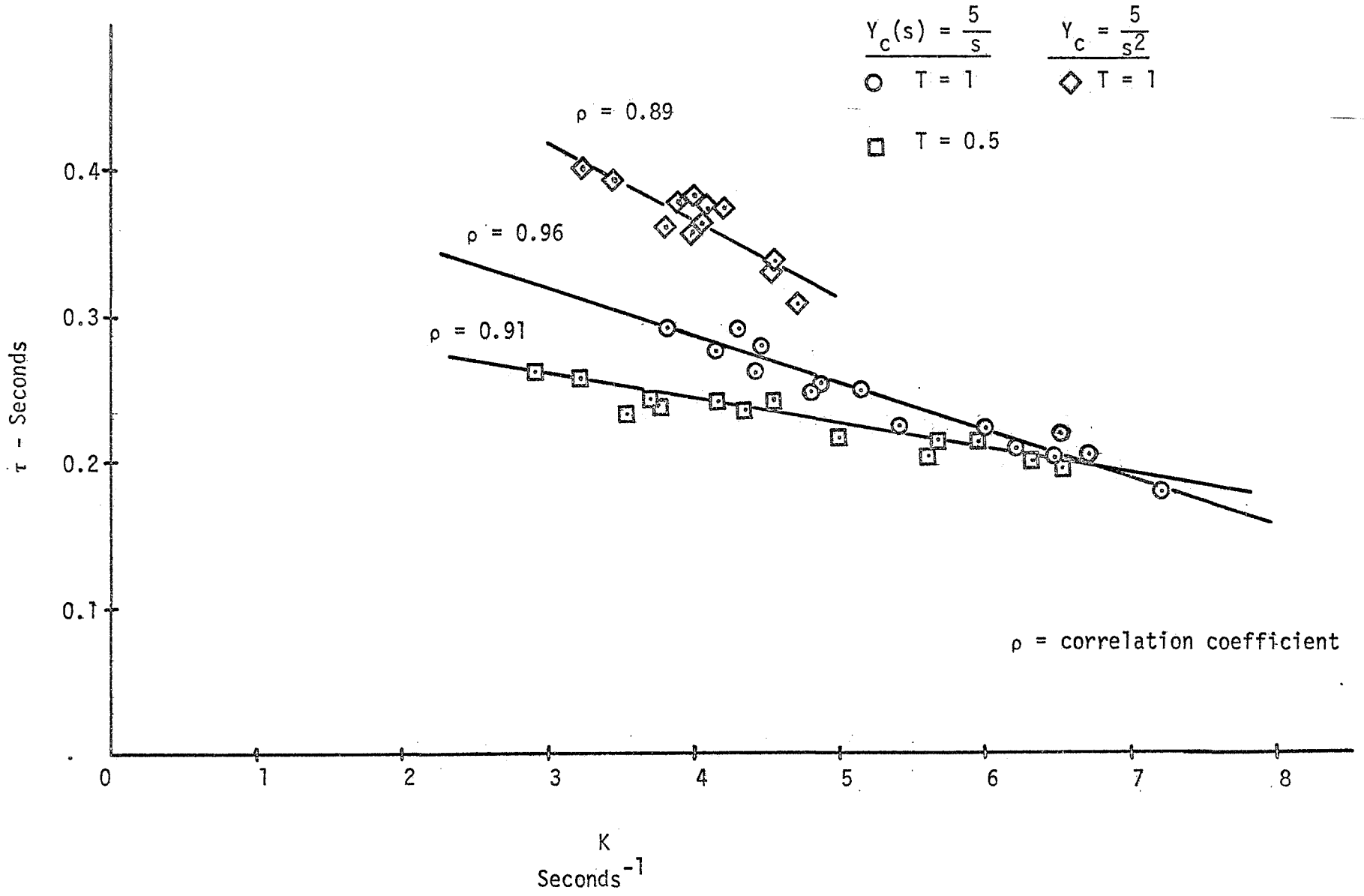



Figure 3 Correlation of K and τ

It is important to re-emphasize at this point that K and $\tau$ are linearly related, in spite of the fact that the data involves different subjects and various intervals in each subject's training period.

In the remainder of this report it will be shown that the correlation existing between K and $\tau$ can be attributed to an apparent paramater optimization of the approximate crossover model. It will be shown that when K and $\tau$ of the model are adjusted to minimize a realistic performance index at a given test condition, the optimum values of K and $\tau$ fall on essentially the same linear regression lines as found in Figure 3.

D. Development of an Optimal Policy

When the compensatory portion of Figure 1 is replaced by the approximate crossover model of Figure 2, the normalized mean squared control system error (NMSE) for the model is as shown in Figures 4 and 5 for input filter time constants of 1 and 1/2 seconds, respectively. Each set of curves has NMSE as a function of K with $\tau$ as a parameter. Superimposed on each of these sets of curves is the location of the associated K-$\tau$ regression line from Figure 3. From a casual study of Figures 4 and 5 it is not evident that the subjects have adjusted K and $\tau$ to optimize NMSE in any recognizable manner.

To account for the correlation existing in the experimental values of K and $\tau$, three postulates are developed.

Rationale for Postulate 1: At any stage of training, each subject undoubtedly has the capability of operating the closed loop compensatory system at some maximum bandwidth. Further, the closed loop bandwidth can be approximated by the natural frequency of the compensatory system.

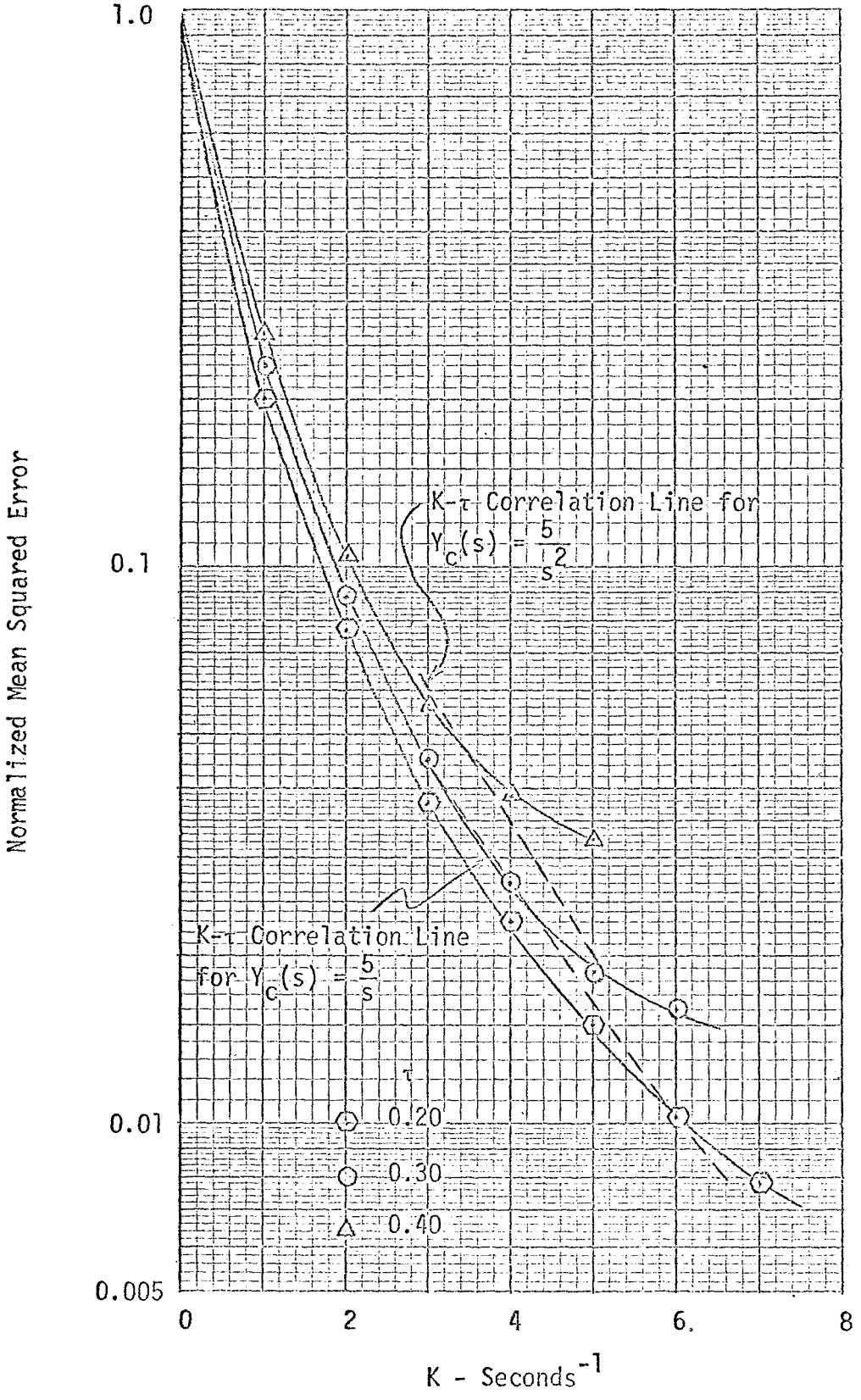



Figure 4 Normalized Mean Squared Error for the Approximate Crossover Model with an Input Filter of $1/(s + 1)^3$.

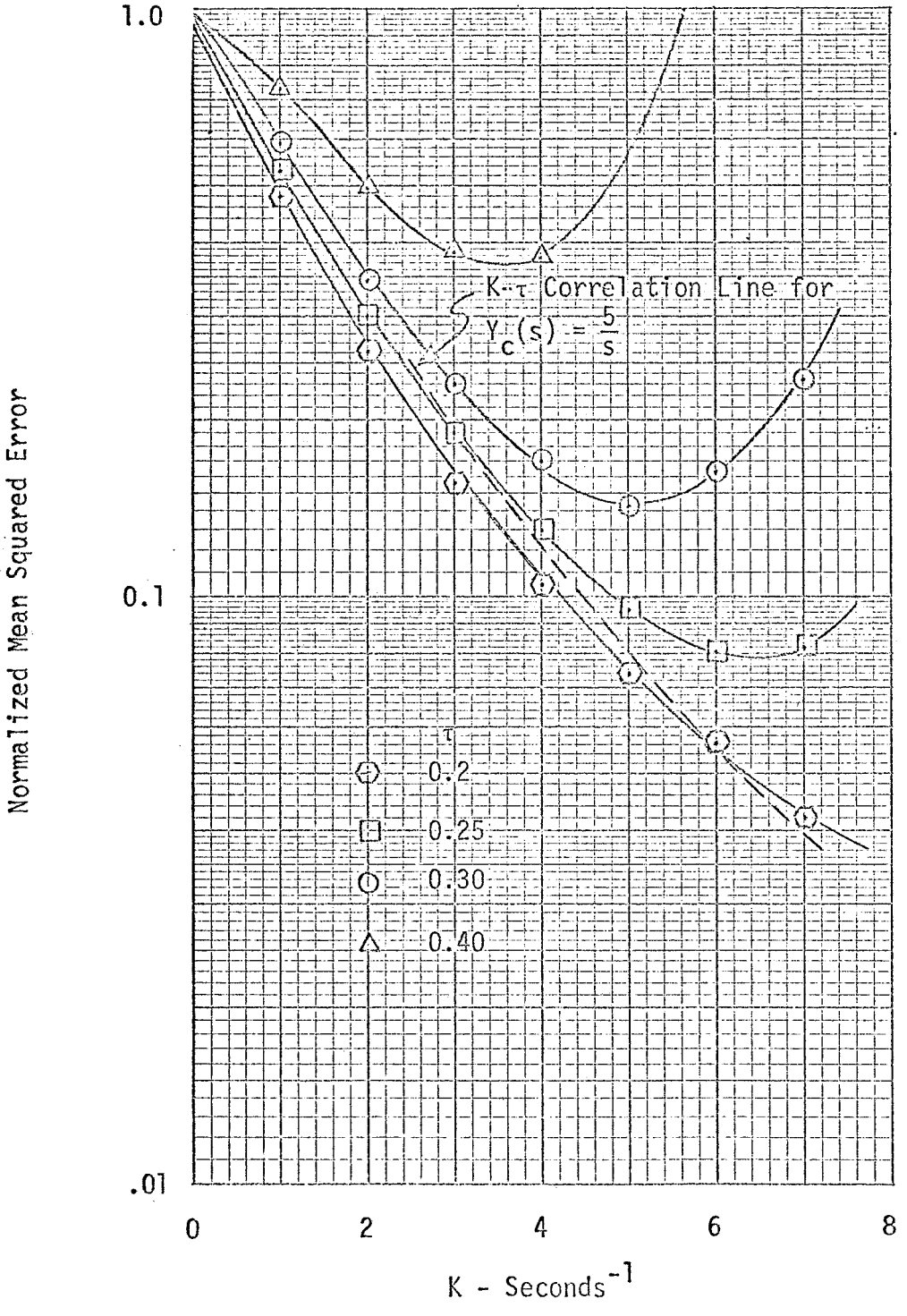



Figure 5 Normalized Mean Squared Error for the Approximate Crossover Model with an Input Filter of $1/(\frac{1}{2}s + 1)^3$.

This approximation is reasonable since the input power at frequencies above the natural frequency is extremely low [8]. In terms of the crossover model the closed loop natural frequency, $\omega_n$, is

$$\omega_n = \sqrt{\frac{2K}{\tau}} \quad .$$

If it is assumed that each subject runs the compensatory system at his maximum bandwidth, and that this maximum is fixed during a given interval of his training, then

$$\sqrt{\frac{2K}{\tau}} = \text{constant} = B = \text{Bandwidth in radians/second.}$$

$$\frac{2K}{\tau} = B^2$$

$$\tau = \frac{2}{B^2} K$$

or

$$\tau = \beta K \quad . \tag{1}$$

Under the conditions assumed, Equation (1) can be considered as a constraint on any optimal adjustment of the parameters K and $\tau$ which might be taking place. It should be emphasized that "$\beta$" in Equation (1) will be different for different subjects, and different for the same subject during various intervals of his training period. Emperically, $\beta$ can be determined to be in the range $0.01 \leq \beta \leq 0.12$ seconds$^2$.

Postulate 1: Any parameter optimization of the approximate crossover model (compensatory control system) is constrained by Equation (1): $\tau = \beta K$, $0.01 \leq \beta \leq 0.12$. $\beta$ is fixed for a given subject, at a given point in training, on a given tracking task.

Rationale for Postulate 2: A general performance index in optimal control theory, and one that seems intuitively reasonable for the human operator, involves the combination of control system error and control effort. Using the notation of Figure 1, one form of this index is

$$J = E\left\{\lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \left[e^2(t) + \alpha c^2(t)\, dt\right]\right\}$$

or

$$J = \overline{e^2} + \alpha \overline{c^2} \tag{2}$$

where $E\{\cdot\}$ is the statistical average, $\overline{e^2}$ is the mean squared value of the tracking error, $e(t)$, and $\overline{c^2}$ is the mean squared value of the controlled variable, $c(t)$. $\alpha$ is a constant.

Postulate 2: The performance index being minimized by the human operator is Equation (2): $J = \overline{e^2} + \alpha \overline{c^2}$ .

Postulate 3: The control strategy of the human operator while controlling one of the elements discussed in Section C is a two-step process consisting of:

(1) Forcing the compensatory system into the form of the approximate crossover model.

(2) Minimizing the performance index

$$J = \overline{e^2} + \alpha \overline{c^2}$$

via a parameter adjustment that is constrained by the relation

$$\tau = \beta K.$$

It should be noted that part (1) of Postulate 3 has already been proven, while part (2) has not.

E. Analysis of the Postulated Strategy

To evaluate the optimal strategy postulated above, and to determine the value of $\alpha$ for each test condition, a digital computer program was developed to generate J for the approximate crossover model. This was done for the following conditions:

Condition 1: $Y_c(s) = 5/s$ and $T = 1$ second.

Condition 2: $Y_c(s) = 5/s$ and $T = 1/2$ second.

In these cases the transfer function relating c(t) to e(t) must be

$$\frac{c}{e}(s) = \frac{K(2/\tau - s)}{5(2/\tau + s)}$$

so that the entire forward loop remains as the approximate crossover model.

Condition 3: $Y_c(s) = 5/s^2$ and $T = 1$ second.

Here,

$$\frac{c}{e}(s) = \frac{Ks(2/\tau - s)}{5(2/\tau + s)} .$$

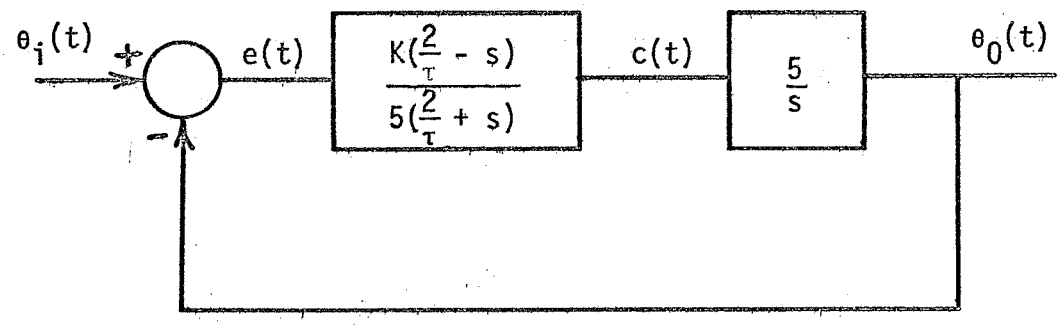
so the approximate crossover model is retained. Figures 6a and 6b show the compensatory system models for the three conditions.

In terms of the block diagrams of Figure 6, the performance index can be reduced to the following integral forms.

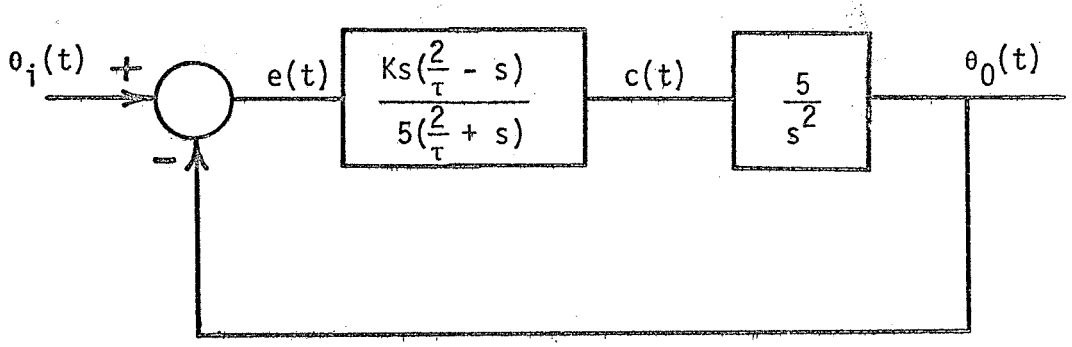
$$J = \overline{e^2} + \alpha \overline{c^2}$$

$$= \frac{1}{2\pi j} \left\{ \int_{-j\infty}^{j\infty} \left| \frac{\theta_i}{N}(s) \right|^2 \left| \frac{e}{\theta_i}(s) \right|^2 \Phi_{NN}(s)\, ds \right.$$

$$\left. + \alpha \int_{-j\infty}^{j\infty} \left| \frac{\theta_i}{N}(s) \right|^2 \left| \frac{c}{\theta_i}(s) \right|^2 \Phi_{NN}(s)\, ds \right\} .$$




(a) $Y_c(s) = \frac{5}{s}$




(b) $Y_c(s) = \frac{5}{s^2}$

Figure 6 Compensatory System Models for Two Controlled Elements Based on Approximate Crossover Model

$\Phi_{NN}(j\omega)$ is the power spectral density of the signal N(t). Assuming $\Phi_{NN}(j\omega) = 1$,

$$J = \frac{1}{2\pi j}\left\{\int_{-j\infty}^{j\infty}\left|\frac{1}{(Ts+1)^3}\right|^2 \cdot \left|\frac{s(2/\tau + s)}{s^2 + (2/\tau - K)s + 2K/\tau}\right|^2 ds \right.$$

$$\left. + \alpha \cdot \int_{-j\infty}^{j\infty}\left|\frac{1}{(Ts+1)^3}\right|^2 \cdot \left|\frac{Ks/5(2/\tau - s)}{s^2 + (2/\tau - K)s + 2K/\tau}\right|^2 ds\right\} \qquad (3)$$

when $Y_c(s) = 5/s$ (Conditions 1 and 2), and

$$J = \frac{1}{2\pi j}\left\{\int_{-j\infty}^{j\infty}\left|\frac{1}{(Ts+1)^3}\right|^2 \cdot \left|\frac{s(2/\tau + s)}{s^2 + (2/\tau - K)s + 2K/\tau}\right|^2 ds \right.$$

$$\left. + \alpha \cdot \int_{-j\infty}^{j\infty}\left|\frac{1}{(Ts+1)^3}\right|^2 \cdot \left|\frac{Ks^2/5(2/\tau - s)}{s^2 + (2/\tau - K)s + 2K/\tau}\right|^2 ds\right\} \qquad (4)$$

when $Y_c(s) = 5/s^2$ (Condition 3). These equations were also normalized by dividing by $\overline{e^2}$ at K = 0. The normalization factor happens to be 3/16T in all cases, under the assumption that N(t) is a white noise source with unity power level. The solution of these equations was accomplished by use of tabulated solutions of Parseval's Theorem [13], under the constraint, $\tau = \beta K$.

A programmed search of the solutions to Equations (3) and (4) was made in the following manner. With $\alpha$, $\beta$ and T fixed, $\min_{K,\tau} J$ was determined. This was done for various values of $\alpha$, $\beta$ and T in order to find that value of $\alpha$ which, for a given $Y_c(s)$ and T, would yield K and $\tau$ values equivalent to those present on the subject regression lines in Figure 3. If the postulated strategy is correct, and if all subjects are minimizing the same functional at a given test condition, then the optimum values of K

and $\tau$ should fall on the regression lines of Figure 3, regardless of the value to which $\beta$ is fixed. This was indeed shown to be the case.

The results of the computer analysis may be summarized as follows:

For Condition 1: $Y_c(s) = 5/s$ and $T = 1$ second, $\alpha = 2.5$.

The $\min_{K,\tau} J$ with $J = \overline{e^2} + 2.5\overline{c^2}$ and $\tau = \beta K$, where $\beta$ is any fixed value $0.01 \leq \beta \leq 0.08$, occurs at values of K and $\tau$ which fall essentially on the subject's $Y_c(s) = 5/s$, $T = 1$ regression line of Figure 3.

For Condition 2: $Y_c(s) = 5/s$ and $T = 1/2$ second, $\alpha = 0.25$.

The $\min_{K,\tau} J$ with $J = \overline{e^2} + 0.25\overline{c^2}$ and $\tau = \beta K$, where $\beta$ is any fixed value $0.01 \leq \beta \leq 0.08$, occurs at values of K and $\tau$ which fall essentially on the subject's $Y_c(s) = 5/s$, $T = 1/2$ regression line of Figure 3.

For Condition 3: $Y_c(s) = 5/s^2$ and $T = 1$ second, $\alpha = 0$.

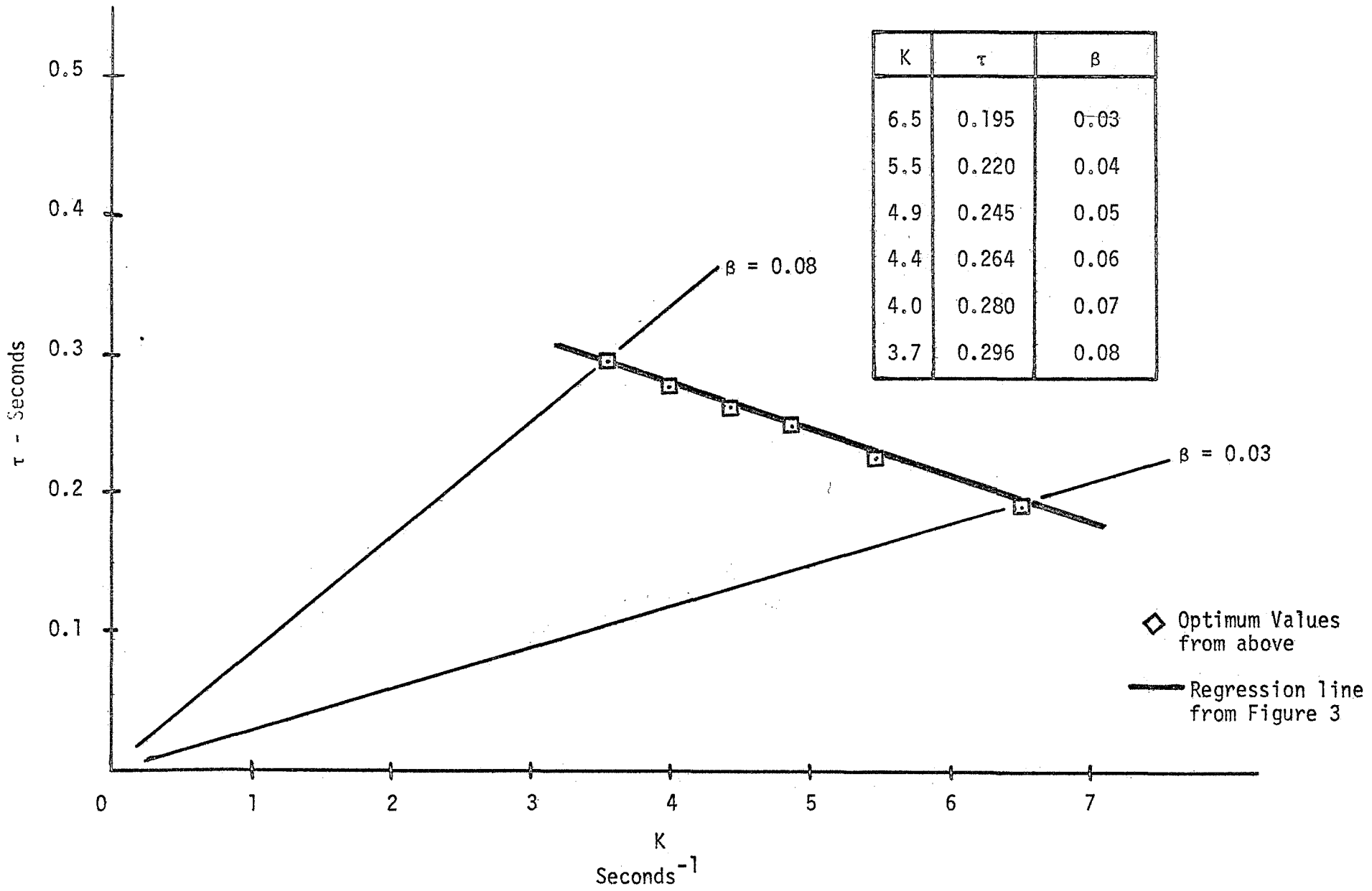
The $\min_{K,\tau} J$ with $J = \overline{e^2}$ and $\tau = \beta K$, where $\beta$ is any fixed value $0.07 \leq \beta \leq 0.12$, occurs at values of K and $\tau$ which fall essentially on the subject's $Y_c(s) = 5/s^2$, $T = 1$ regression line of Figure 3.

The optimum values of K and $\tau$ for several values of $\beta$ are given in Figures 7, 8 and 9 for Conditions 1, 2 and 3, respectively. These figures indicate the strong relationship between the K-$\tau$ regression lines of Figure 3 and the optimum K-$\tau$ values derived from the postulated performance index.

## F. Conclusions and Extensions

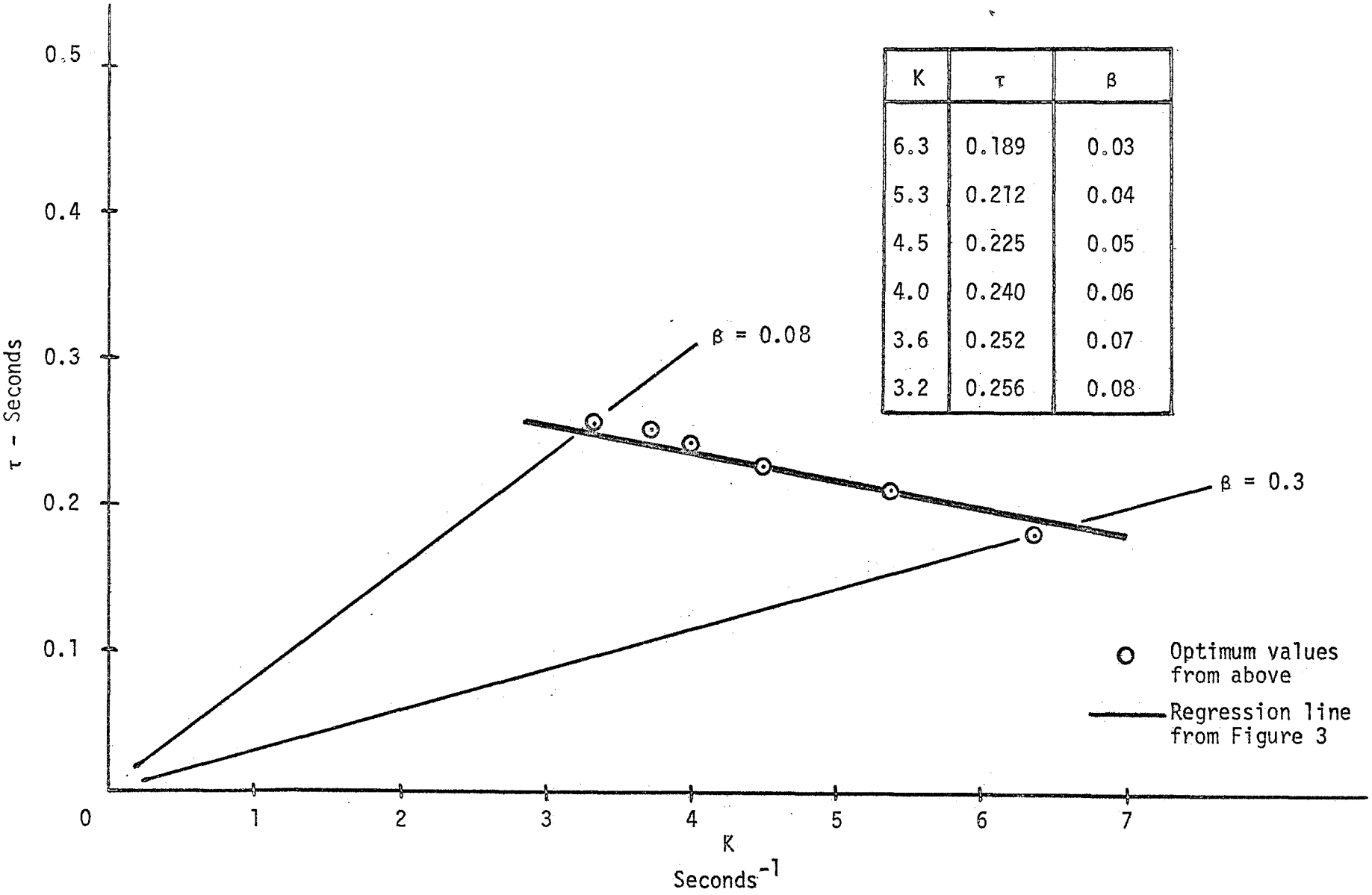
### Conclusions

From the data summarized in Figures 7-9 several tentative conclusions can be drawn.



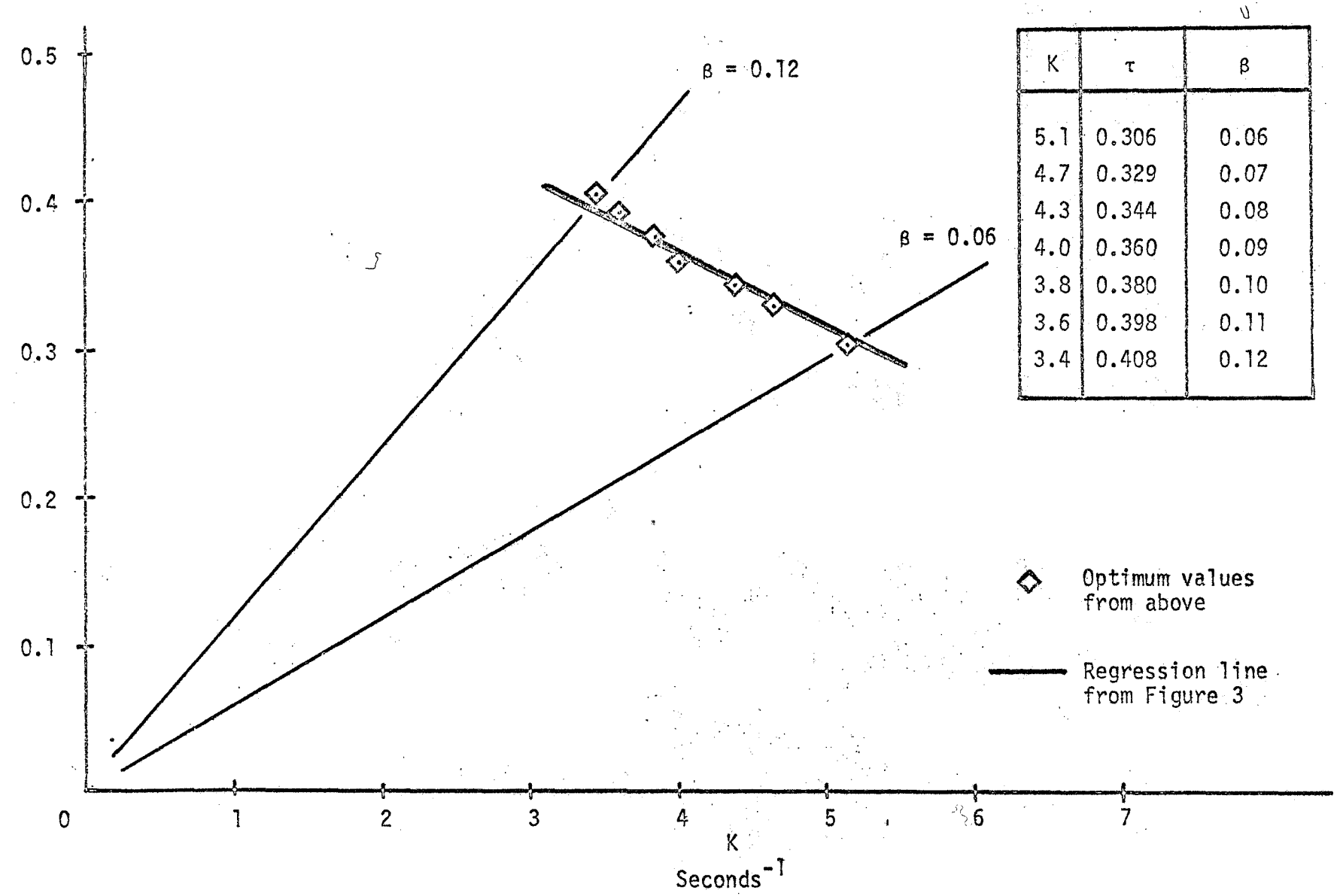

| K | τ | β |
|---|---|---|
| 6.5 | 0.195 | 0.03 |
| 5.5 | 0.220 | 0.04 |
| 4.9 | 0.245 | 0.05 |
| 4.4 | 0.264 | 0.06 |
| 4.0 | 0.280 | 0.07 |
| 3.7 | 0.296 | 0.08 |

Figure 7 Optimum K and τ Values, J = MSE + 2.5(MSC), $Y_c(s) = \frac{5}{s}$ , T = 1




| K | τ | β |
|---|---|---|
| 6.3 | 0.189 | 0.03 |
| 5.3 | 0.212 | 0.04 |
| 4.5 | 0.225 | 0.05 |
| 4.0 | 0.240 | 0.06 |
| 3.6 | 0.252 | 0.07 |
| 3.2 | 0.256 | 0.08 |

Figure 8 Optimum K and τ Values, J = MSE + 0.25(MSC), $Y_c(s) = \frac{5}{s}$ , T = 0.5




| K | τ | β |
|---|---|---|
| 5.1 | 0.306 | 0.06 |
| 4.7 | 0.329 | 0.07 |
| 4.3 | 0.344 | 0.08 |
| 4.0 | 0.360 | 0.09 |
| 3.8 | 0.380 | 0.10 |
| 3.6 | 0.398 | 0.11 |
| 3.4 | 0.408 | 0.12 |

Figure 9 Optimum K and τ Values, J = MSE, $Y_c(s) = \frac{5}{s^2}$, T = 1

(1) The postulated optimum strategy yields crossover model parameters which are in close agreement with the data obtained from six subjects and two controlled elements which were available for analysis. Thus, the hypothesis that subjects in compensatory control systems essentially minimize the performance index $J = \overline{e^2} + \alpha\overline{c^2}$ for their associated crossover model, under the constraint $\tau = \beta K$, appears valid and warrants further experimental analysis.

(2) For a given test condition a unique value of $\alpha$ in the performance index $J = \overline{e^2} + \alpha\overline{c^2}$ exists which is the same for all subjects. The $\beta$'s in the constraint $\tau = \beta K$, however, vary between subjects. The conclusion can be drawn that all subjects minimize the same performance index, but arrive at different parameter values due to the subject's varying abilities in controlling higher frequency components.

(3) Test Conditions 1, 2 and 3 are arranged in order of increasing task difficulty. The value of $\alpha$ shows a definite tendency to decrease as task difficulty increases. This implies that subjects penalize control effort more on easier tasks than they do for harder tasks. They can "keep the error small" and still conserve control energy when the task is easy. However, as the task becomes harder this is not true, and they thus trade off control effort in favor of reducing the system error, which is the assigned task.

Extensions:

(1) Since only six subjects were used in obtaining the experimental data from which the postulates were drawn and evaluated, several additional subjects should be run to see if their model coefficients fall on the same regression lines as found for the first subjects. This extension has been started, and is making extensive use of both the equipment purchased under this grant, and Oakland University's new Hybrid computer facility.

(2) If the postulates are correct, the changes in the values of K and $\tau$ for a given trained subject, due to small variations in the spectrum of the input signal, should be predictable. An interesting area of research would be to see if this prediction can indeed be made.

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## H. Appendix

### Subject Tracking Data

Each K and $\tau$ value in the following tables was determined by averaging over those K-$\tau$ values determined in 5 two-minute tracking tasks. These experimental tests are discussed at length in references [8,9].

Table 1

$Y_c(s) = 5/s$

| Subject No. | Day of Training | Input Filter Time Constant (Seconds) | K ($seconds^{-1}$) | $\tau$ (seconds) |
|---|---|---|---|---|
| 1 | 2 | 1.0 | 4.92 | 0.240 |
| | 2 | 0.5 | 3.63 | 0.221 |
| | 4 | 1.0 | 5.23 | 0.240 |
| | 4 | 0.5 | 4.61 | 0.234 |
| | 6 | 1.0 | 5.52 | 0.215 |
| | 6 | 0.5 | 5.08 | 0.201 |
| | 8 | 1.0 | 6.55 | 0.194 |
| | 8 | 0.5 | 5.70 | 0.198 |
| | 10 | 1.0 | 6.79 | 0.198 |
| | 10 | 0.5 | 6.05 | 0.206 |
| 2 | 2 | 1.0 | 4.95 | 0.245 |
| | 2 | 0.5 | 4.24 | 0.231 |
| | 4 | 1.0 | 6.10 | 0.216 |
| | 4 | 0.5 | 5.75 | 0.206 |
| | 6 | 1.0 | 6.61 | 0.210 |
| | 6 | 0.5 | 6.30 | 0.205 |
| | 8 | 1.0 | 7.28 | 0.172 |
| | 8 | 0.5 | 6.39 | 0.192 |
| | 10 | 1.0 | 6.31 | 0.202 |
| | 10 | 0.5 | 6.62 | 0.188 |
| 3 | 2 | 1.0 | 3.93 | 0.282 |
| | 2 | 0.5 | 3.01 | 0.249 |
| | 4 | 1.0 | 4.42 | 0.281 |
| | 4 | 0.5 | 3.33 | 0.246 |
| | 6 | 1.0 | 4.25 | 0.266 |
| | 6 | 0.5 | 3.75 | 0.233 |
| | 8 | 1.0 | 4.50 | 0.253 |
| | 8 | 0.5 | 4.45 | 0.234 |
| | 10 | 1.0 | 4.54 | 0.273 |
| | 10 | 0.5 | 3.83 | 0.229 |

Table 2

$Y_c(s) = 5/s^2$

| Subject No. | Day of Training | Input Filter Time Constant (Seconds) | K ($seconds^{-1}$) | τ (seconds) |
|---|---|---|---|---|
| 4 | 3 | 1.0 | 3.32 | 0.392 |
| ↓ | 5 | ↓ | 4.12 | 0.347 |
| ↓ | 7 | ↓ | 3.90 | 0.353 |
| ↓ | 9 | ↓ | 4.80 | 0.301 |
| 5 | 3 | 1.0 | 4.19 | 0.365 |
| ↓ | 5 | ↓ | 4.30 | 0.367 |
| ↓ | 7 | ↓ | 4.14 | 0.357 |
| ↓ | 9 | ↓ | 4.65 | 0.329 |
| 6 | 3 | 1.0 | 3.56 | 0.384 |
| ↓ | 5 | ↓ | 4.03 | 0.369 |
| ↓ | 7 | ↓ | 4.10 | 0.373 |
| ↓ | 9 | ↓ | 4.64 | 0.325 |

## III. GRANT ACTIVITIES

Publication: The results of this research will be submitted for presentation at the 6th Annual Manual Control Conference to be held in March 1970. After the extensions to this research have been completed, the material will be submitted for publication in the IEEE Transactions on Man-Machine Systems.

Personnel Supported by the Grant

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